J10Q.2 - Angular Momentum

Problem

A two-particle system is in a state $|\Psi_0\rangle$, where each particle has orbital angular momentum quantum numbers $\ell = 1$ and $m_\ell = 0$.
Let $\vec{L}_{\rm tot} = \vec{L}_1 + \vec{L}_2$ be the total angular momentum of the two particles, where $L^2_{\rm tot}$ has eigenvalues $\hbar^2 L(L+1)$.

a) If the two-particle state is expanded in eigenstates of $L^2_{\rm tot}$, which values of L have non-zero amplitude in the expansion? For each of these values, what is the probability that it will be found in a measurement of $|\vec{L}_{\rm tot}|^2$?

At time $t = 0$, a coupling between the particles is "switched on", so that for $t > 0$ the time evolution of the state is governed by the Hamiltonian

$$H = \gamma \vec{L}_1 \cdot \vec{L}_2 \,.$$

The amplitude $f(t) = |\langle \Psi(t) | \Psi_0 \rangle|^2$ oscillates as a function of time, returning to the value 1 at times $t = t_n = nT$. What is the period T?

b) What is the value of $f(t)$ when $t = (t_n + t_{n+1})/2$?