J10T.2 - Maxwell-Boltzmann Gas

Problem

In a simple approximation often used to calculate transport properties, the statistical distribution of the velocities of molecules arriving at a point is taken to be that of the *local* equilibrium state where their most-recent collision occurred ($T, p, \langle\vec{v}\rangle$, *etc.*, are assumed to be slowly varying functions of position).

a) Using this approximation, derive the well-known estimate (due to Maxwell) of the viscosity η of a dilute classical gas of molecules with mass m, particle density $\bar{n}$, and mean free path ℓ between collisions. Assume a Maxwell-Boltzmann distribution of molecular velocities with $\langle|\vec{v} - \langle\vec{v}\rangle|^2\rangle = v_{\text{rms}}^2$.

b) If ℓ is modeled by treating monoatomic molecules as hard spheres with a finite diameter, how does the predicted viscosity vary with pressure p for low pressures at fixed temperature T? (Assume that ℓ remains smaller than the dimensions of the container.)

The Maxwell-Boltzmann gas can be viewed as the high-temperature limit of a quantum gas of non-relativistic particles. The Maxwell-Boltzmann treatment assumes that

$$\lambda(T) \ll \bar{n}^{-1/3} \ll \ell \,,$$

where $\lambda(T)$ is the *thermal de Broglie wavelength* of the particles.

c) In terms just of the three lengths $\lambda(T), \bar{n}^{-1/3}$, and ℓ, plus fundamental constants, give expressions for:

   i. The viscosity η of a Maxwell-Boltzmann gas.

   ii. The entropy density $\bar{s}$ of a monoatomic Maxwell-Boltzmann gas.

d) Estimate the lowest value that the ratio $\eta/\bar{s}$ can take before the quantum effects neglected in Maxwell-Boltzmann theory must be considered.