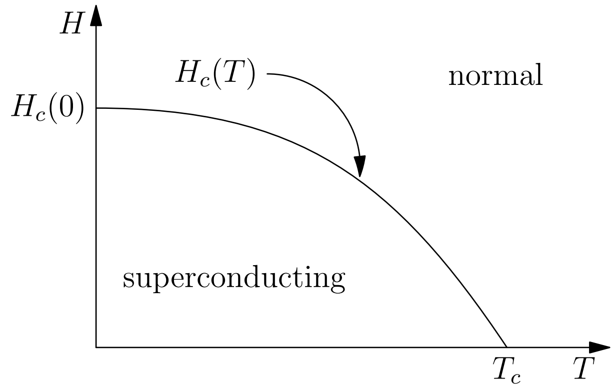
J10T.3 - Thermodynamics of Superconductors (O85T.1)

Problem



In the absence of a magnetic field ($H = 0$), an isotropic metal has a continuous transition to a superconducting state below a critical temperature T_c. The metal has specific heat (per unit volume) $c_V^n = \gamma T$, while in the superconductor $c_V^s = \alpha T^3$. Assume that the volume of the material does not vary with the temperature and magnetic field.

a) Find T_c as a function of γ and α.

b) For $H = 0$, give expressions in terms of T, T_c, and γ for (and sketch versus T):

i. the free energy density,

ii. the entropy density

iii. the specific heat.

In finite magnetic field strength $H > 0$, the transition becomes first order. The superconductor exhibits the Meissner effect which excludes magnetic flux density B from its interior, so $B = 0$ even though $H > 0$. Above a critical field $H_c(T)$, superconductivity breaks down, and the system becomes normal with $B = \mu H$ (to a good approximation, μ is equal to the vacuum permeability). The phase diagram is depicted above.

c) On general grounds, why must $dH_c(T)/dT$ vanish as $T \to 0$?

d) Find an expression for $H_c(T)$. (Assume that c_V^n and c_V^s do not depend on H.)

Note: When the internal energy U is defined to include the integrated electromagnetic energy density inside the material, H is a thermodynamic analog of the pressure:

$$H \equiv V^{-1}\, \partial U/\partial B|_{SVN}\,.$$