Section B. Electricity and Magnetism

1. Birefringent material

A "birefringent" (doubly-refracting) crystalline material is a material with an anisotropic dielectric tensor, so its index of refraction depends on the plane of polarization of incident electromagnetic waves.

For a certain material, the dielectric tensor is given by

$$\epsilon_{ij} = \epsilon \begin{pmatrix} 1 & \Delta & 0 \\ \Delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where ϵ, Δ are real, and $0 < \Delta < 1 - \epsilon^{-1}$, which means that the constitutive relations of the material are

$$D_i = \epsilon_0 \sum_j \epsilon_{ij} E_j, \quad B_i = \mu_0 H_i,$$

where ϵ_0 and μ_0 are the vacuum permittivity and permeability.

(a) Find the two possible frequencies $\omega_\nu(k)$ and corresponding polarization directions (of $\vec{E}_0$) of a plane wave

$$\vec{E} = \vec{E}_0 \cos(kz - \omega_\nu(k)t)$$

propagating parallel to the z-axis inside the material with a given wavevector $\vec{k} = (0, 0, k)$.

(b) The plane $z = 0$ is the surface between the vacuum ($z < 0$) and the birefringent material ($z > 0$). A plane wave with wavenumber $\vec{k} = (0, 0, k_0)$, frequency ω, and with linear polarization along the x-axis, coming from the vacuum, is normally incident on the surface of the birefringent material. What is the direction of polarization of the reflected wave?

Note: you may find it convenient to change to a different Cartesian coordinate system in which the dielectric tensor is diagonal.