Section A. Mechanics

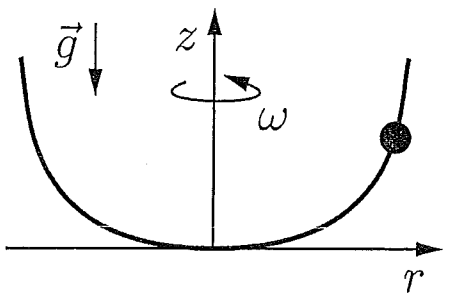
1. Bead sliding on a rotating wire

A bead with mass m slides without friction on a "U-shaped" wire which lies in a plane parallel to the z-axis, and follows a path

$$z = \frac{r^4}{a^3}$$

where r is the distance from the z-axis to a point on the wire at height z. The downwards acceleration g due to gravity is parallel to the z-axis.

The wire is now constrained to rigidly rotate about the z-axis with constant angular frequency ω.



(a) Derive the equation of motion of the bead in terms of its radial distance $r(t)$ from the z-axis.

(b) Find the equilibrium points r_{eq} of the radial motion (*i.e.*, motion where $r(t) = r_{\text{eq}}$, constant),

(c) Find the frequencies of small radial oscillations about those equilibrium points that are stable.