

Section B. Statistical Mechanics and Thermodynamics

1. One-dimensional model for rubber

To model rubber, consider a polymer of N molecules of length a , connected end to end. One end of the molecular chain is fixed at $x = 0$. Assume that the molecules can only be oriented parallel to the x -axis, and that all possible configurations of the chain have the same internal energy. The chain is kept at fixed temperature T by contact with a heat-bath.

In this model, the two ends of molecule n are on the x -axis, at x_{n-1} and x_n , with $x_n - x_{n-1} = \pm a$, and $x_0 = 0$.

- (a) What is the entropy of the chain when the free end is at $x_N = L$?
- (b) What is the tension in the chain when it is stretched to length L , with $L \ll L_{\max} = Na$?
- (c) For N large, how much work is required to stretch the chain from its equilibrium state with unconstrained length to a length $L \leq L_{\max}$?
- (d) During the stretching process, does the heat-bath absorb or give out heat? (Explain your answer).