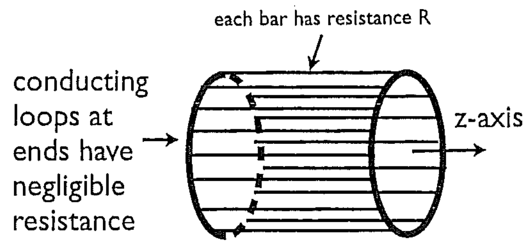
3. **Torque produced by an AC induction motor.**

An induction motor consists of two components. The "*stator*" produces a time-dependent magnetic field with a direction that rotates with angular frequency ω_s, determined by the frequency of the AC current source. Take it to be

$$(B_x(t), B_y(t), B_z(t)) = B_0(\cos\omega_s t, \sin\omega_s t, 0).$$

The second component is the *rotor*, which in one design resembles a cylindrical "cage" that is free to rotate about its axis (the z-axis), formed by $N \gg 1$ equally-spaced conducting bars of length ℓ, each with resistance R, connected by a metal ring of radius r at each end of the cylinder (which has negligible resistance).



Electrical currents in the cage may be described as N independent current loops, each one defined by two adjacent bars, connected by the rings at each end of the cage. Assume the self-inductance and mutual inductances of these loops are negligible (much smaller than R/ω_s). The positions of the bars are defined by the line segments

$$(x, y, z) = (r\cos\theta_j, r\sin\theta_j, z), \quad 0 < z < \ell, \quad j = 1, 2, \ldots, N.$$

with $\theta_j = 2\pi j/N + \omega_r t$ (the rotor rotates with angular velocity $\omega_r \neq \omega_s$).

(a) As a function of time, what is the induced emf across bar j?

(b) Find the time-averaged torque exerted on the rotor as a function of ω_r.