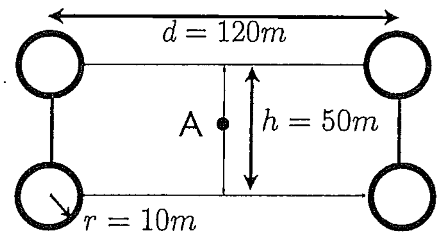
3. **"Space hotel" (rotating space station with artificial gravity)**

In the design of a space station to be used as a "space hotel" for astronauts to live in, it is proposed to make it spin to produce an "artificial gravity" effect.

The "hotel" consists of four "living-quarter pods" which can each be modeled as a hollow spherical shell with a mass $m = 3\times10^4$kg and a radius $r = 10$m. These are coupled together with rigid beams of negligible mass so that their centers are at the vertices of a $d \times h = 120\text{m}\times50\text{m}$ rectangle.



(a) Compute the moments of inertia I_1, I_2, I_3 (relative to the center of mass A of the structure) for the three orthogonal principal axes of the moment-of-inertia tensor, chosen so that $I_1 < I_2 < I_3$. Identify these three axes on a sketch of the "hotel". (Note that $5^2 + 12^2 = 13^2$.)

In the design, the "hotel" rotates about the I_2 principal axis with a frequency f_g so that an astronaut at the center of a pod experiences the equivalent of 1/5 of the Earth's gravity at sea level. This "centrifugal force" is directed away from the axis of rotation.

(b) What is the value of f_g required to generate this apparent gravity?

The "hotel" is constructed, and made to rotate as described above. Unfortunately, the connecting beams are slightly flexible, which allows damped small vibrational modes of the structure to become excited and dissipate rotational kinetic energy as heat, while conserving the angular momentum $\vec{L}$ of the rotating hotel.

(c) Estimate the characteristic time scale for the exponential growth of ω_3, the angular frequency of rotation about the axis with moment of inertia I_3. Explain why the rotational motion about the axis with moment of inertia I_2 is not stable with respect to dissipation of the total rotational kinetic energy.