Section A. Quantum Mechanics

1. (Quantum Zeno effect) A two-level quantum system is represented as a spin-$\frac{1}{2}$ entity, evolving under the Hamiltonian

$$H \;=\; \hbar\, \vec{\boldsymbol{B}} \cdot \vec{\sigma}$$

with $\vec{\boldsymbol{B}} = \{b, 0, 0\}$ pointing in the x direction, and $\vec{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\}$ the Pauli spin matrices:

$$\boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The system is prepared in the state $\sigma_z = +1$ at time $t_0 = 0$.

The following questions concerns the results of measuring the quantity σ_z at later times, assuming the measurement itself does not change the value of σ_z (though of course it cannot preserve the relative phase between the states $|\sigma_z = 1\rangle$ and $|\sigma_z = -1\rangle$).

a) What is the probability of finding the system in the flipped state, $\sigma_z = -1$, at time $t_1 > 0$?

b) The measurement is repeated at time $t_2 = t_1 + \tau$, after recording the value of σ_z at time t_1 (either $\sigma_z = +1$ or $\sigma_z = -1$). What is the probability that the system will be found at the state $\sigma_z = -1$ at t_1?

c) Next, re-starting from the state $\sigma_z = +1$ the observer begins to measure σ_z repeatedly at intervals $\tau \ll b^{-1}$, up to time T ($\gg \tau$), that is at times $t_n = n\tau$ with $n = 1, 2, 3, ..., T/\tau$. How small should τ be made in order to reduce the probability of finding the system at time T in the flipped state ($\sigma_z = -1$) to less than $p_0 = 0.1$?