Section B. Statistical Mechanics and Thermodynamics

1. (Atmosphere's 'adiabatic lapse rate')

   This problem is intended to estimate how rapidly the atmosphere cools as you climb a mountain, modeling the atmosphere by an ideal gas which is in equilibrium with respect to adiabatic convection. The gas is of molar weight M (that is, 1 mole of the gas has the mass of M grams), and its molecules are of f degrees of freedom. At the surface of the earth ($z = 0$) the gas is at pressure P_0 and temperature T_0; the gravitational acceleration is g and the ideal gas constant is R.

   a) Derive the relationship between the temperature T of the gas and the pressure P, for an adiabatic flow of the gas (of f degrees of freedom per molecule).

   b) Assuming the gas is in mechanical equilibrium (that is, at each height z there is no net force on a slab of the gas of thickness dz) derive $\frac{dP}{dz}$.

   c) Use parts (a) and (b) to compute the dry adiabatic lapse rate $\frac{dT}{dz}$ (the change in temperature with altitude) in terms of M, g, R and f.

   d) Derive an expression for the pressure P as a function of the height z, for an adiabatic atmosphere.