

2. (Hydrogen's two spin isomers)

Molecular hydrogen H_2 consists of two protons and two electrons. Protons are fermions with spin = $1/2$. The spins of the two electrons are paired so that the net electronic spin is 0, but the nuclear spins of the protons can be in either the total $S = 1$ spin state or the $S = 0$ state.

a) Write down the partition function for the two nuclear spin states of H_2 . Assume the energy difference between the $S = 1$ and $S = 0$ states is ΔE , with $S = 0$ the lower one.

b) At $T = 300$ K, hydrogen is 70% $S = 1$ and 30% $S = 0$. Calculate a value for ΔE from this data.

c) The latent heat ΔL of hydrogen at its boiling point (20.3 K at 1 bar) is 445 kJ/kg. The conversion of $S = 1$ to $S = 0$ H_2 is very slow (measured in days), and it is quite possible to quickly cool hydrogen gas to its boiling point with the 300 K nuclear spin distribution intact. Assume $\Delta E = 10^{-21} J$. Determine if liquid H_2 will entirely boil off when the spin conversion to the ground state occurs. Assume perfect thermal insulation, and that the hydrogen's nuclear spins convert completely to their ground state.