3. The Hamiltonian for the spin degrees of freedom of Positronium in a magnetic field in the z-direction is given by

$$H \;=\; \alpha\, \vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)} + \beta\, (\sigma_z^{(1)} - \sigma_z^{(2)})\,, \tag{1}$$

where α and β are constants and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the vector of Pauli matrices (listed in problem QM1).

a) As a warm-up: describe the degeneracy of the energy levels of the Hamiltonian at $\beta = 0$.

b) Calculate the *eigenvalues* and describe (to the extent that you can without writing 'messy' expressions) the *eigenvectors* of H.