Section B. Statistical Mechanics and Thermodynamics

1. Consider a relativistic gas of N indistinguishable non-interacting spin-1/2 fermions of zero rest mass, initially at equilibrium in a (three-dimensional) volume V_i at zero temperature, $T_i = 0$. These are fictitious massless fermionic particles that have no antiparticles and have energy $\epsilon(\vec{p}) = c|\vec{p}|$, where $\vec{p}$ is the particle's momentum.

   a) Calculate the initial total energy E_i of this zero-temperature relativistic Fermi gas.

   b) The initial confining walls are then instantaneously removed and this gas expands into a vacuum to a much larger final volume V_f (enclosed by thermally insulating walls), and then internally equilibrates due to weak (and particle-number-conserving) interactions between the fermions. V_f is so large that quantum statistics can be ignored, and the final state of the gas can be treated as "classical", although still relativistic. What is the final temperature T_f of this gas?

   c) What was the change in the entropy ΔS of the gas due to this expansion?