2. A paper towel roll consists of thin towels with total mass m, linear mass density λ, and negligible thickness, wrapped around a thin hollow tube of mass M and radius R. The tube's central axis, about which it rotates without friction, is horizontal. The free end of the first towel is at $x = 0$ initially and the corresponding point on the tube is at $\theta = 0$. Initially, the towels wrap a precise integer number of times around the tube so that the mass density is uniform around the tube. Then the roll is rotated clockwise by an arbitrarily small angle so that a short segment of towel separates from the roll (see figure) and the roll is thus no longer in mechanical equilibrium. The roll then starts to rotate clockwise from rest as the towels unwind from the roll. Assume that the unwound towels are exactly vertical and never touch another surface. Also assume that the towels do not slip with respect to each other or with respect to the tube, and neglect any dissipation anywhere in this system.

   What is the angular velocity of the roll as a function of θ?