2. Consider N classical two-state spins $S_i = \pm 1$, with Hamiltonian

$$H = -\frac{J}{N}\sum_{i=1}^{N}\sum_{j=1}^{i-1} S_i S_j - h\sum_{i}^{N} S_i \ .$$

This is an "infinite-range" model where all spins interact with all others: the spin-spin coupling (J/N) is defined so the energy J remains a positive constant in the thermodynamic limit $N \to \infty$. You should work in this limit. The external magnetic field h can be of either sign or be zero.

a) Sketch the equilibrium phase diagram of this system vs. temperature T and field h, showing all phase transitions and critical points that occur as one varies T and/or h.

b) Calculate the critical temperature T_c.

c) At the critical temperature $T = T_c$, calculate the the magnetization $m = \langle S_i \rangle$ as a function of the field h in the limit where $|h|$ is small but nonzero.