3. An upright cylindrical bucket has radius R and its rim is at height H. This bucket is placed on a horizontal surface and filled to a height $h < H$ with incompressible water. The bucket is then rotated at angular frequency ω about a vertical axis that goes through the center of the bucket. Let g be the acceleration due to gravity, and assume that the water is simply rotating with the same angular velocity as the bucket.

   a) Assuming that ω is small enough that the water does not reach the rim of the bucket or reveal the bottom of the bucket, find an expression for the height of the water's surface as a function of the radial distance from the central axis of rotation.

   b) Quantitatively, what are the conditions on ω such that water neither spills over the rim of the bucket nor reveals the bottom of the bucket?