3. Consider a gas with pressure

$$p(T, V) = aT^x \ ,$$

where a is a constant and the exponent x satisfies $x > 1$. Note that the equilibrium pressure of this gas does not depend on its volume V, only on its temperature T. Assume this gas has total energy $E(T, V = 0) = 0$ at $V = 0$ for all T, and has entropy $S(T = 0, V = 0) = 0$.

(a) What familiar textbook system could this be? What is the exponent x in that case? For general a and general $x > 1$, obtain the entropy $S(T, V)$ of this gas at equilibrium for all $T \geq 0$ and all $V \geq 0$.

Consider a reversible heat engine with this gas (for general a and general $x > 1$) as the working medium: Each cycle starts at volume V_A and temperature T_2. First isothermally expand the gas to volume V_B while in contact with the hot reservoir with temperature T_2. Then remove the gas from contact with the reservoirs and expand adiabatically until the temperature drops to T_1, the temperature of the cold reservoir ($T_1 < T_2$). Complete the cycle by compressing the gas first isothermally at T_1, then adiabatically, to return to the start of the next cycle.

(b) Sketch this cycle in the pV plane. Give the equations for $p(V)$ along all parts of the cycle. Make sure your sketch in the pV plane is qualitatively accurate.

(c) Show that the efficiency of this reversible heat engine is equal to the standard Carnot result.