Section B. Statistical Mechanics and Thermodynamics

1. Approximate the daily cycle of temperature T as a function of time t at the Earth's surface where the air is in contact with the ground as

$$T(t) = T_0 - T_1 \cos(\omega t) \ .$$

Assume the solid Earth below this flat horizontal surface is a homogeneous material with temperature-independent specific heat per unit volume c and thermal conductivity κ. We use the standard definition of thermal conductivity so the heat current density is given by

$$\vec{j}_Q = -\kappa \vec{\nabla} T \ .$$

a) Calculate the temperature $T(t, z)$ at time t and depth z below the surface.

b) Considering the daily cycle, assume the minimum and maximum air temperatures occur at 4 AM and 4 PM. At what times of day is the heat current zero at the surface (no heat flow between the air and the ground)?

c) If the daily and yearly oscillations of air temperature both have the same amplitude T_1 (not true in Princeton, but it is roughly true at various locations), what is the ratio of the characteristic depths to which these two temperature signals penetrate the Earth?