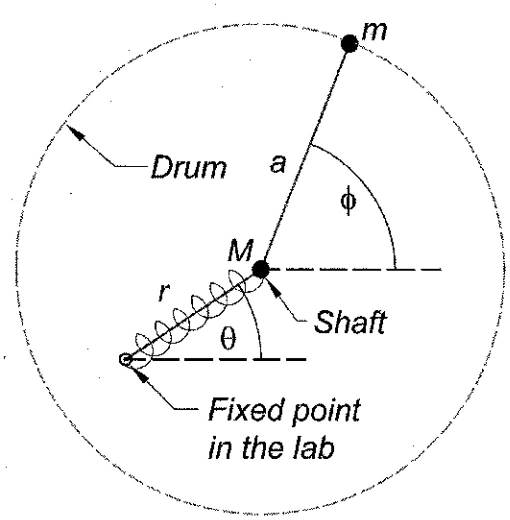
2. **Washer.** Consider a top-loading washing machine consisting of a vertical cylindrical drum of mass M and moment of inertia I around an axis along its shaft, about which it rotates. The washing machine load is not balanced, and we approximate this asymmetric load as a single added point mass m located at a fixed radius a from the shaft, and at a fixed azimuth relative to the drum. The figure shows a top view of the washer.

   The shaft remains vertical, but otherwise its position is not fixed in the lab (laundromat) frame. Rather, a set of massless springs with effective force constant k connect the shaft to a fixed support, taken to be the origin of the system. The position of the shaft is then described by cylindrical coordinates (r, θ) in the lab frame, and the springs have effective relaxed length zero ($r = 0$ is the static equilibrium position). The azimuth of the line from the shaft to the mass m is labeled ϕ, and the time derivative of ϕ is constrained to be constant by the washing machine motor, so $\phi(t) = \Omega t$. The purpose of the spring system is to stabilize the motion of the drum in the presence of an unbalanced load.



   a) Write down the equations of motion for r and θ.

   b) There is a finite and stable steady-state solution to these equations with $\dot{r} = \ddot{r} = \ddot{\theta} = 0$ for all Ω, except right on resonance. Find these solutions for Ω both below and above the resonance.