3. A system of two massive particles, of spins $s_a > 0$ and $s_b > 0$, is governed by the Hamiltonian:

$$H = K + V(|\vec{r}_a - \vec{r}_b|) + f(|\vec{r}_a - \vec{r}_b|)\left(S_z^{(a)} - S_z^{(b)}\right) ,$$

with K the usual kinetic energy operator and $\vec{S}^{(a)}$ and $\vec{S}^{(b)}$ the spin operators. The function f is negative at all distances: $f(r) < 0$, and the interaction potential $V(r)$ is finite and sufficiently attractive so that the system has at least one bound state. Let $\vec{S} = \vec{S}^{(a)} + \vec{S}^{(b)}$ be the total spin operator.

a) Explain why this system's ground state is non-degenerate.

b) What are the ground-state *expectation values* of the total spin's component S_z, and of the total spin operator $|\vec{S}|^2$? For which of these operators is the ground state also an eigenfunction?

c) Consider now the case $s_a = 1$, $s_b = 1/2$. List the possible values of $|\vec{S}|^2$ and S_z. What are the probabilities of observing these outcomes when $|\vec{S}|^2$ and S_z are measured in the system's ground state?