3. Consider a classical one-dimensional magnet with Hamiltonian

$$H = -J \sum_{i=1}^{N} \vec{S}_i \cdot \vec{S}_{i+1}$$

where each $\vec{S}_i$ is a classical (3-component) vector spin of fixed length S.

a) Calculate $\langle \vec{S}_i \cdot \vec{S}_{i+1} \rangle$ at equilibrium at temperature T.

b) Calculate the specific heat per spin $c(T)$ of this system in the limit $N \to \infty$.

c) Consider the $T \to 0$ limit of part b). Is this consistent with the behavior of $c(T)$ for a **quantum** ferromagnet ($J > 0$) of spin S with this same Hamiltonian? If not, estimate (roughly) and state the correct quantum behavior of $c(T)$ for small T, explaining your reasoning. Ferromagnetic spin waves in this model have a frequency that depends on wavenumber k as $\omega(k) \sim k^2$ for small k.