Section C. Quantum Mechanics

1. A quantum particle moves in one dimension with energy as a function of wavenumber $E(k)$. Its momentum is $p = \hbar k$ and is conserved. At time $t = 0$ the wavefunction $\psi(x, t = 0)$ of this particle is a minimum-uncertainty wavepacket centered at the origin ($x = 0$) in real space and with average momentum $\langle p \rangle_{t=0} = \hbar k_0$. Assume that the initial uncertainty in the position $\sqrt{\langle x^2 \rangle_{t=0}} = \sigma$ is large but finite, so the uncertainty in the momentum is small but nonzero. Thus approximate $E(k)$ by its Taylor expansion about k_0 keeping terms only to order $(k - k_0)^2$.
(a) In terms of the given parameters; $E(k_0)$; and $\frac{dE}{dk}$ and $\frac{d^2E}{dk^2}$ evaluated at $k = k_0$, obtain the normalized wavefunction $\psi(x, t)$ at nonzero times t. Do not make any assumption about the dispersion relation $E(k)$ other than that its first and second derivatives exist and are finite at k_0.

(b) Calculate the expectation values: $\langle x \rangle_t$, $\langle p \rangle_t$, $\langle (x - \langle x \rangle_t)^2 \rangle_t$ at nonzero times t.
[If you get bogged down: first do this problem assuming $\frac{d^2E}{dk^2} = 0$ before letting it be nonzero.]