Section D. Thermo and Statistical Mechanics

1. Consider a long molecule like DNA that is made out of two polymer strands, with links connecting the monomers between one strand and the other; let the number of links be N. Now imagine that we grab the ends of the two strands and pull them apart with a force F. In order to lengthen the segment that we are pulling on, we have to break links. Each time we break a link the energy of the molecule goes up by an amount Δ, the "bond energy" of each link.
On the other hand, each time we break a link, the ends we are pulling on move apart by a distance $2l_\circ$, where $l_\circ$ is the distance between the links along one strand. Thus, the energy of the molecule with n links broken is $E(n) = n(\Delta - 2Fl_\circ)$.

(a) Find an equation that relates the mean number of broken links $\langle n \rangle$ at temperature T to the partition function Z.
(b) Define $a = \left(\frac{\Delta - 2Fl_\circ}{k_B T}\right)$. Evaluate the partition function Z. Show that in the limit of large N, the behavior of Z is very different depending on whether F is smaller or larger than a "critical" value F_c. What is value of and the physical meaning of F_c?
(c) Use your result for Z to calculate $\langle n \rangle$ in terms of a in the limit of large N.