2. **Higgs Orbits**. A particle of mass m moves under the influence of a potential

$$V(r) = -ar^2 + br^4, \tag{1}$$

where a and b are positive constants and r is the distance between the particle and the force center. In the context of quantum field theory with r being a complex scalar field, this is the so-called "Higgs Potential." Here we interpret the potential as arising from a central force and investigate the possible motion of a particle in this field.

a) What is the radius ρ of the circular orbit allowed in this potential?

b) What is the condition on a and b such that this orbit is stable?

c) What is the frequency of small oscillations around $r = \rho$?

In the Higgs analogy, the answer to part a corresponds to the vacuum expectation value of the field, while the answer to part c corresponds to the mass of the Higgs boson. The stability of the vacuum (part b) was used as early as 1976 to set a lower bound on the mass of the Higgs boson.