

2. Consider two indistinguishable nonrelativistic **bosons** of mass  $m$ , constrained to move one-dimensionally around a circle of perimeter  $L$ . The particles each have **spin-1**, and they interact via a spin-independent potential that is a Dirac delta-function:  $V(x_1, x_2) = g\delta(x_1 - x_2)$ , where  $x_i$  is the position on the circle (in arc length) of particle  $i$ .

(a) First look at zero interaction,  $g = 0$ , being careful to only include states of the correct symmetry for these indistinguishable spin-1 bosons. What are the energies and the degeneracies of the ground state and of the lowest-energy excited state? In each case, say what value(s) of total spin these states may have.

(b) Add a weak interaction  $g \neq 0$ . Now what are the degeneracies of the ground state and of the lowest-energy excited state? For each sign of  $g$ , say what value(s) of total spin these states may have.

(c) Solve for a two-particle ground state wavefunction, including showing the spin state. Do this first at  $g = 0$ , and then all other  $g \neq 0$ . In the latter case you may leave one parameter in the wavefunction specified only as the solution to an equation that you will not be able to solve analytically.