2. A recent experiment on trapped atomic gases by the Zwierlein group at MIT reports the data shown in the figure. In this problem you will try to gain an understanding of a part of the data (the highlighted curve) which is well described by a free Fermi gas.

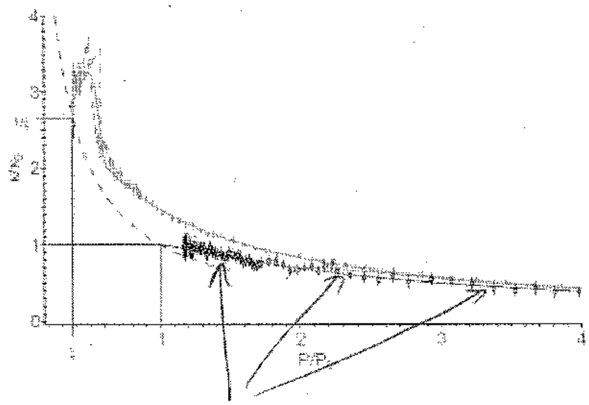

Figure 1: Normalized compressibility versus normalized pressure from Ku et al., *Science* 335, 563 (2012).

a) The isothermal compressibility of a finite volume of gas is defined as

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T,N}$$

Show that in the infinite volume limit this reduces to

$$\kappa = \frac{1}{n}\left(\frac{\partial n}{\partial p}\right)_{T}$$

where $n = N/V$.

b) Consider a three dimensional gas of spinless fermions of mass m and density n at $T = 0$. Calculate its pressure $p_0(n)$ and compressibility $\kappa_0(n)$.

c) Now consider the gas at a non-zero temperature T. Let us define

$$\tilde{p}(n,T) = \frac{p(n,T)}{p_0(n)}$$

and

$$\tilde{\kappa}(n,T) = \frac{\kappa(n,T)}{\kappa_0(n)}$$

which are the quantities plotted in the figure.
Use dimensional analysis to show that $\tilde{\kappa}$ can be written as a function $\kappa(\tilde{p})$ of $\tilde{p}$ alone.

d) By construction, $\tilde{\kappa}(1) = 1$. Calculate the leading asymptotic behavior of $\tilde{\kappa} - 1$ as $\tilde{p} \to 1$. [Useful result: $p(n,T) = p_0(n)\{1 + \frac{5\pi^2}{12}(\frac{k_B T}{\epsilon_F(n)})^2 + \cdots\}$]