3. A metal has two phases, N (normal) and S (superconducting). Assume that in the normal phase the magnetization M (per unit volume) due to an applied external magnetic field H is negligible, so the magnetic induction or flux density $B \equiv \mu_0(H+M) = \mu_0 H$ in the normal metal phase.

The metal is cooled down to a temperature T in a large magnetic field H, and then H is reduced to zero. At temperatures $T < T_c$, it is observed that as H is reduced there is a critical field

$$H_c(T) = H_0 \left[1 - \left(\frac{T}{T_c}\right)^2\right],$$

where a first-order phase transition from the normal state to the superconducting state occurs. The magnetic flux is completely expelled from the metal, and $B = 0$ in its interior (Meissner effect) for $H < H_c(T)$ in the superconducting state.

Recall that the magnetic variant of the Gibbs free energy (per unit volume) is $G(T, H) = U - TS - M'H$, where $M' = \mu_0 M$ in SI units. (Ignore any thermal expansion of the metal, and treat its volume as fixed).

(a). Find the difference of the entropy densities $\Delta S(T) = S_N(T) - S_S(T)$ between the normal and the superconducting phases (with the assumption of negligible magnetization in the normal phase, $S_N(T)$ is independent of H).

(b). If the system is heated in the absence of a magnetic field (at $H = 0$) it undergoes a continuous (second-order) phase transition from superconductor to normal metal at $T = T_c$. What is the discontinuity in its specific heat per unit volume at this phase transition? (Make a sketch showing how the specific heat varies with temperature near the transition). Which phase has the larger specific heat?

(c). By how much is the ground-state (T=0) energy per unit volume of the superconductor lower than that of the normal metal, when $H = 0$?