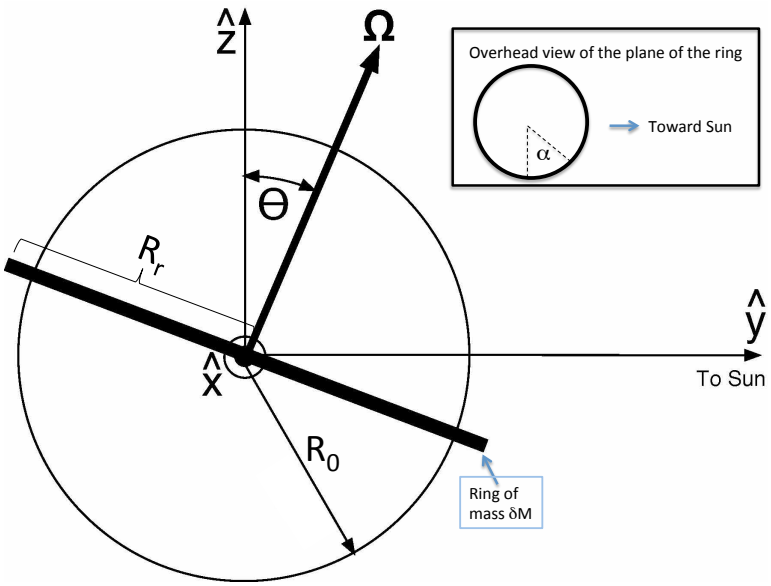
2. This is the dawning of the age of Aquarius, due to the precession of the Earth's spin axis $\vec{\Omega}$ around the celestial orbital axis $\hat{z}$. The Earth is slightly elliptical due to its spin. Approximate the Earth as a perfect sphere of radius R_0 and mass M, but assume a thin ring of radius R_r with mass δM is in the plane of the equator of the Earth. The Sun has mass M_s and is a distance $R_{\rm es}$ from the center of the Earth to the Sun. Assume that $R_r \ll R_{es}$, $\dfrac{R_r - R_0}{R_0} \ll 1$, and $\delta M \ll M$.



(a) From the figure, what is the torque $\vec{\tau}$ acting on the Earth about its center of mass due to the Sun? You'll need to do some approximations to get a tractable answer. One approximation is to use only the y coordinate in estimating how far each point on the ring is from the Sun.

(b) Neglecting the effects of the Moon's gravity, what is the rate of precession ω_p of the angular momentum $\mathbf{L}$ of the Earth around the celestial axis? If you couldn't solve part (a), then assume the torque is $\vec{\tau}$, where you don't need to know the magnitude of the vector $|\vec{\tau}| = \tau_p$, but you should know the direction. You may take the magnitude of the angular momentum, L, as known.