2. Scattering from a spherical potential:
(a) Calculate the differential cross-section, $d \sigma / d \Omega$, for a particle of mass $m$ scattering from a spherical potential $V(r)=V_{0} e^{-(r / a)^{2}}$ using the first-order Born approximation. You may need the integral

$$
\int_{0}^{\infty} \sin r e^{-(r / b)^{2}} r d r=\frac{\sqrt{\pi}}{4} b^{3} e^{-b^{2} / 4}
$$

(b) Calculate the total cross-section. It may be helpful to use the representation $\left|\vec{k}-\vec{k}^{\prime}\right|=2|\vec{k}| \sin (\theta / 2)$, where $\theta$ is the angle between $\vec{k}$ and $\vec{k}^{\prime}$.
(c) What are the conditions on $V_{0}, a$ and/or $k$ for the first-order Born approximation to be valid?

