Section B. Statistical Mechanics and Thermodynamics

1. (**The weather underground**) Assume the Earth is flat with its surface at $z = 0$. The solid material below the surface has a temperature-independent thermal diffusivity D. The weather above ground is a highly regular climate with sinusoidal annual (a) and daily (d) oscillations of the temperature, so the temperature at the surface as a function of time t is

$$T(t, z = 0) = T_o + T_a \cos(\omega_a t) + T_d \cos(\omega_d t) \ .$$

   Assume the temperature at infinite depth below ground ($z \to -\infty$) is T_o.

   (a) What is the temperature $T(t, z)$ below ground at time t and position $z < 0$?

   (b) What is the position z closest to the surface where the annual temperature cycle is opposite to that at the surface (so that below Princeton it is instead "hottest" in January and "coldest" in July)?

   (c) By what factor is the annual temperature variation attenuated at the above depth?