3. This problem explores some elements of a mass spectrometer. Parts (a) and (b) may be answered independently and treated non-relativistically.

   (a) An ion of charge $+q$ and mass m is accelerated through a potential V_o as shown in Fig. 3(A). It enters a region between two very long (in the direction perpendicular to the page) cylindrical electrodes of radius a and b respectively. Find the potentials $V(a)$ and $V(b)$ such that the ion moves in a circle of radius r_o.

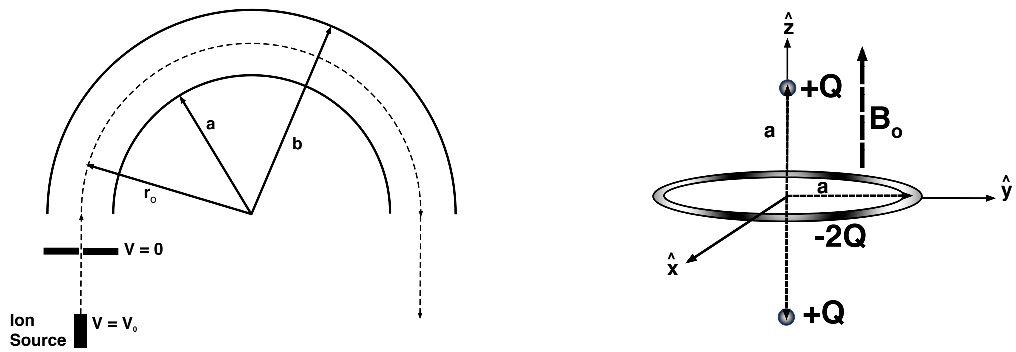

Figure 3: (A) Electrostatic filter. (B) Penning trap.

   (b) A Penning trap is used in Fourier-transform mass spectrometry. At the simplest level, the Penning trap consists of a uniform magnetic field $B_o\hat{z}$ and a quadrupole electric field. Let us assume that the electric field is generated by two positive charges $+Q > 0$ located at $\pm a\hat{z}$ and a uniformly charged ring of radius a and charge $-2Q$ centered around the origin in the xy-plane, as shown in Fig. 3(B). This setup is rotationally symmetric around the z-axis.

      i. Close to the origin, the electric field takes the form

$$\mathbf{E} = k_z z\hat{z} + k_r r\hat{r} \, ,$$

      where $r = \sqrt{x^2 + y^2}$. Determine k_z and k_r.

      While the general motion of an ion of mass m and charge $q > 0$ in the Penning trap is quite complicated, here we investigate only two particular cases:

      ii. Find the frequency ω_z of small oscillations around the origin in the case where the ion moves only along the z-axis.

      iii. Assume the ion moves *uniformly* along a circle of radius $R \ll a$ in the plane at $z = 0$. What is the angular frequency for this motion? Interpret the answer in the limit of large B_o.