3. (a) Consider the Hamiltonian for a general time-independent, one-dimensional potential $V(x)$,

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\,.$$

Show that for an arbitrary, continuous function $\phi(x)$, the value of

$$E = \frac{\langle\phi|H|\phi\rangle}{\langle\phi|\phi\rangle}$$

gives an upper bound on the ground state energy for the potential $V(x)$.

(b) For a particle moving in a triangular potential well

$$V(x) = \begin{cases} \infty & \text{if } x < 0\,, \\ V_0 x/L & \text{if } x > 0\,, \end{cases}$$

the energy levels take the form

$$E_n = \alpha_n V_0 \left(\frac{\hbar^2}{mL^2V_0}\right)^q\,,$$

where α_n and q are numerical constants. Determine the value of the exponent q.

(c) Using the approach in part (a), find an estimate for the constant α_0 corresponding to the ground state in the triangular potential well. (The estimate needs not be optimal, but should be based on a reasonable variational calculation.)