3. (**An ideal Bose gas)**

Consider an ideal Bose gas of particles with zero spin and the dispersion relation

$$E_{\vec{k}} = c\,|\vec{k}|\,.$$

(a) Show that at high temperatures the mean energy per particle for this gas satisfies

$$U/N = \alpha T$$

with a constant α, and determine the value of that constant (in three dimensions).

Hint: a useful relation is $\int_0^\infty dx\, x^\nu\, \beta e^{-x\beta} = \nu \int_0^\infty dx\, x^{\nu-1}\, e^{-x\beta}$ (for $\beta, \nu > 0$).

(b) The pressure of such a gas with N particles becomes independent of N below a critical temperature , T_c. Explain why and calculate this temperature.

(c) Calculate the specific heat c_V of the gas for $T < T_c$.

(d) Does the transition occur if such a gas is confined to two spacial dimensions?

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