Section B. Electricity and Magnetism

Radiating charge

1. A magnetic field $\mathbf{B}(x)$ points in the $\hat{z}$ direction in half of space as follows.

$$\mathbf{B}(x) = 0 \quad \text{for} \quad x < 0 \tag{1}$$
$$\mathbf{B}(x) = B_0\hat{z} \quad \text{for} \quad x > 0 \tag{2}$$

There is no electrostatic field. A non-relativistic electron of mass m_e initially moves at constant velocity $\mathbf{v} = v_0\hat{x}$ at $y = -a$ in the xy plane in the space with $x < 0$. As shown in the figure, at $t = 0$ the electron enters the space $x > 0$, travels in a semi-circle of radius a with an angular frequency of $\omega_0 = eB_0/m_e$ for a time π/ω_0 and then exits the region of field with $\mathbf{v} = -v_0\hat{x}$ at $y = a$.

(a) What is the differential power per unit solid angle, $dP(t)/d\Omega$, that an observer on the x-axis at a distance $x = r\hat{x} >> c/\omega_0$ measures when observing the charge? Sketch $dP(t)/d\Omega$ as a function of time and quantitatively label the duration, amplitude, and key features. [Hint: A lot of progress can be made using a simple model of how a charge radiates.]

(b) For an observer on the z-axis at a distance $z = r\hat{z} >> c/\omega_0$, again sketch the $dP(t)/d\Omega$ he or she would measure as a function of time. Quantitatively label the duration, amplitude, and key features.

(c) What is the total energy emitted by the electron?