2. A point particle of mass m and electric charge q moves in a 3d harmonic oscillator potential with frequency ω and a uniform electric field of strength $\mathcal{E}$ pointing in the z-direction. The Hamiltonian is

$$H = \frac{\vec{p}^2}{2m} + \frac{m\omega^2\vec{r}^2}{2} - q\mathcal{E}z\,.$$

(a) What are the eigenenergies of this Hamiltonian?

(b) Find an expression for the ground state wave-function.

Assume now that the system is described by the above Hamiltonian only for $t < 0$, and that at $t = 0$ the electric field is suddenly turned off. For $t < 0$, the system is in its ground state.

(c) What is the probability that the system will end up in the new ground state right after the electric field is turned off?

(d) What is the expectation value of the electric dipole moment $\vec{d} = q\vec{r}$ at some given time $t > 0$?