Unicycle

3. One of the challenges of Jadwin Hall you no doubt have experienced is riding a unicycle down the ramp and not falling over as you do the $180°$ turn midway down the ramp. In this simplified study of unicycle stability, we assume that the total mass of the wheel plus rider is M, we assume the moment of inertia of the rider is zero, that the wheel has mass m, radius ℓ, and moment of inertia $I_w = m\ell^2$ about its axis, and that the center of mass H_{cm} of the unicycle and rider is at the top of the wheel. As you negotiate the turn, the unicycle moves with velocity V in a circular path of radius of $R \gg \ell$.
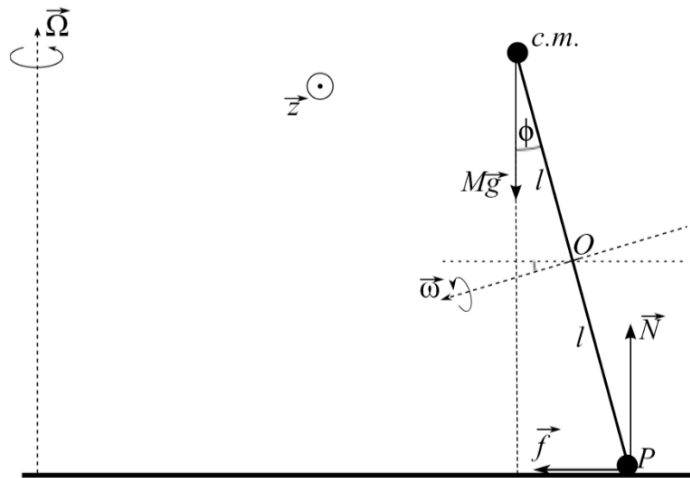

Figure 1: Coordinate system for the problem.

(a) Find the angle ϕ that the unicycle must lean through in the turn.

(b) Put in some typical numbers and and discuss the role of the angular momentum of the wheel in the stability of the unicycle. You can probably do this part successfully without the correct answer to (a).