Section A. Quantum Mechanics

1. **Perturbed Harmonic Oscillator (MR solution)**

   A particle of mass m moves one-dimensionally in a static harmonic oscillator potential

$$V = \frac{1}{2} m \omega^2 x^2$$

   It is also acted on by a space-time dependent perturbation potential $W(x, t)$ that is narrowly localized around a point $x_0(t)$ in space that moves with time. To simulate this, take the delta function expression

$$W = \lambda \delta(x - x_0(t))$$

   where λ parametrizes the potential strength.

   Let $x_0(t) = vt$ for some velocity v and suppose that the particle was in the oscillator ground state $u_0(x)$ in the remote past (at time $t \to -\infty$). What is the probability that the particle will be found in the first excited oscillator state $u_1(x)$ in the remote future? Treat W as a small perturbation and work out the answer to lowest order in λ. Sketch the dependence of the transition probability on v and identify the value of v that maximizes the transition probability.

   You are reminded that

$$u_0 = \left(\frac{1}{\pi a^2} \right)^{1/4} \exp(-x^2/2a^2), \qquad u_1 = \sqrt{2} \frac{x}{a} u_0, \qquad a = \sqrt{\frac{\hbar}{m\omega}}.$$