2. **Long Antenna Pattern**

A thin, straight, conducting wire centered on the origin and oriented along the z-axis carries a current

$$I = \hat{z} I_0 \cos \omega t$$

everywhere along its length ℓ. This antenna will radiate electromagnetic waves with frequency ω and wavelength $\lambda = 2\pi c/\omega$. We will not assume that $\ell << \lambda$.

(a) Because of the current, a time-dependent charge $q(t)$ will accumulate at the two ends of the wire. Give expressions for the charge and current *densities* $\rho(\vec{x}, t)$ and $\vec{j}(\vec{x}, t)$ on the wire. Show that the electric dipole moment of this charge distribution satisfies $p(t) = p_0 \sin(\omega t)$ and evaluate p_0

(b) Use these source densities to construct the scalar and vector potentials everywhere outside the source region ($r \gg \ell$). Do not assume anything about the relative magnitudes of ℓ and λ. Do state the gauge you are using.

(c) Compute the angular distribution of the energy flux radiated from this antenna. Show that it reduces to the standard electric dipole radiation pattern when $\lambda \gg \ell$. For general λ, show that the energy flux radiated perpendicular to the $\hat{z}$ direction depends only on the maximum electric dipole moment p_0 (and agrees with the standard electric dipole radiation result).