2. **Ideal Gas with Funny Dispersion Relation**

Consider a gas of non-interacting particles with no internal degrees of freedom confined to a three-dimensional box of volume V, and obeying the dispersion relation $\epsilon(\vec{k}) = A\,|k|$. We want to study the dilute gas equation of state for this gas, taking account of quantum statistics.

(a) Let $N(\epsilon) = VH(\epsilon)$ be the total number of single particle states in the box with energy less than ϵ. Define the density of states by $dN(\epsilon)/d\epsilon = VG(\epsilon)$. Calculate $H(\epsilon)$ and $G(\epsilon)$ for the given dispersion relation.

(b) Use the grand canonical ensemble to derive the standard expressions for pressure and density :

$$n = \int_0^\infty d\epsilon \frac{G(\epsilon)}{(e^{\beta(\epsilon-\mu)} \mp 1)} \qquad p = \int_0^\infty d\epsilon \frac{H(\epsilon)}{(e^{\beta(\epsilon-\mu)} \mp 1)}$$

where μ is the chemical potential and the minus (plus) signs correspond to Bose (Fermi) statistics.

(c) Show that in the limit of large negative μ (low density) the pressure satisfies the ideal gas equation of state (independent of statistics).

(d) Finally, compute the first non-trivial correction (in powers of density) to the ideal gas equation of state and determine how it depends on density and temperature for both bosons and fermions.