3. **Magnetic Barrier**

Consider the quantum mechanics of a charged particle moving in the $x - y$ plane subject to a magnetic field $B_z = B_0\theta(x)\theta(d-x)$. In other words, the magnetic field is constant in a strip of width d and zero everywhere else. Including a vector potential in the Schrödinger equation is very simple: just replace ∇ in the kinetic energy by $\nabla - \frac{ie}{\hbar c}\vec{A}(\vec{x})$. We will study the problem of scattering from this 'barrier' of an electron incident from $x < 0$ with momentum parallel to the x-axis. Note: you must choose a gauge for the vector potential describing the B field — the gauge in which $A_x = A_z = 0$ everywhere, with only $A_y(x)$ non-vanishing is particularly convenient here.

(a) For an incident wave $\exp(ikx)$ there will, in general, be a transmitted wave $T\exp(i\widehat{k}x)$ and a reflected wave $R\exp(-ikx)$. Show how the transmitted wave vector $\widehat{k}$ is determined by k and B_0d.

(b) For a given choice of B_0d, you will find that, below a certain critical energy E_0, $\widehat{k}$ is imaginary. Show that a classical description of how an electron moves in the presence of a magnetic strip leads to the same critical energy.

(c) When $\widehat{k}$ is real, there is a transmitted wave that propagates to $x = +\infty$. Calculate the transmitted probability flux and show that, despite the fact that the wave function depends only on x, the flux is *not* along the x-axis! Give a classical interpretation of this quantum fact.

(d) Show that in the limit $d \to 0$, $B_0 \to \infty$, with B_0d fixed, this effectively one-dimensional scattering problem can be solved exactly, and find the reflection and transmission coefficients R and T.