3. **Hydrogen Recombination**

In this problem we investigate the formation of hydrogen atoms in the early universe. Although the binding energy of hydrogen is 13.6 eV, the majority of protons and electrons did not become bound into atoms until the temperature of the neutral primordial plasma cooled to a much lower temperature, about 0.3 eV. To study this problem, we make four assumptions:

- The hydrogen atom has no bound states apart from its ground state.
- We ignore other bound complexes that might be formed, e..g, hydrogen ions and molecules.
- All interactions among hydrogen atoms protons and free electrons are ignored (apart from the fundamental process of atom formation).
- Everything is in thermal equilibrium.

There are two questions:

(a) Assume that at $T = 0.3$ eV half of the protons had a bound electron. From this information calculate the densities (in units of particles per cubic centimeter) of free electrons, free protons, and hydrogen atoms.

(b) At $T = 0.3$ eV what is the density of photons? How does the photon density compare to the total density of baryons (protons plus hydrogen atoms) obtained in part (a)? You have just calculated the photon to baryon ration in our universe, starting from a remarkably simple piece of information.

The following constants will be useful to know:

$$hc = 2.0 \times 10^{-25}\,\mathrm{Jm} = 1.3 \times 10^{-6}\mathrm{eVm}, \quad m_{electron} = .5 \times 10^{6}\mathrm{eV}$$

It will also be useful to know the following integral:

$$\int_0^\infty du u^2 \frac{1}{e^u - 1} = 1.4$$